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A METHOD OF SOLVING DETERMINANTS.

By G. Macloskie.

By a simple *Method of Compression* we can reduce a determinant to another of lower order, so as greatly to lessen the labor of its evaluation. This method depends on the following theorem:

$$\begin{vmatrix} a & b & c & \cdot & \cdot \\ e & f & g & \cdot & \cdot \\ j & k & l & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix} = a\ b\ c \begin{vmatrix} 1 & 1 & 1 & \cdot & \cdot \\ \frac{e}{a} & \frac{f}{b} & \frac{g}{c} & \cdot & \cdot \\ \frac{j}{a} & \frac{k}{b} & \frac{l}{c} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

$$= a\ b\ c \begin{vmatrix} \left(\frac{f}{b} - \frac{e}{a}\right) & \left(\frac{g}{c} - \frac{e}{a}\right) & \cdot\ \cdot \\ \left(\frac{k}{b} - \frac{j}{a}\right) & \left(\frac{l}{c} - \frac{j}{a}\right) & \cdot\ \cdot \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot\ \cdot \end{vmatrix} = a \begin{vmatrix} \left(f - \frac{b}{a}e\right) & \left(g - \frac{c}{a}e\right) & \cdot\ \cdot \\ \left(k - \frac{b}{a}j\right) & \left(l - \frac{c}{a}j\right) & \cdot\ \cdot \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot\ \cdot \end{vmatrix}.$$

We have thus reached a new determinant of a lower order which, when multiplied by one of the elements of the original, is equal to the original determinant.

We can obtain this result directly from the original by the following general rule:

$$\begin{array}{|cccc|} a^* & \underline{b} & \underline{c} & \cdot\ \cdot \\ \underline{e} & f & g & \cdot\ \cdot \\ \underline{j} & k & l & \cdot\ \cdot \\ \cdot & \cdot & \cdot & \cdot\ \cdot \\ \hline & -\frac{b}{a} & -\frac{c}{a} & \cdot\ \cdot \end{array}$$

Select any of the elements as the *chief element* (as a in the margin): and mark it and all the other elements of its row and column. Form *compressors* by dividing the chief element into the other elements of its own row, writing each quotient, with the sign changed, under its proper column. Then calling the marked elements of the column to which the chief element belongs *co-compressors*, the required result is got by the following

Rule.—By taking as elements of a new determinant the remaining (unmarked) *elements of the original determinant, each increased by the product of its compressor and its co-compressor, we obtain a determinant of lower order, which, when multiplied by the chief element, is equal to the original determinant.*

NOTES. (1) If a line drawn diagonally from the original position of the chief element strikes the anterior or the upper margin at an *even* position, reckoning from the upper-anterior corner, we must prefix a minus sign to the new determinant: not so when such line strikes an *odd* marginal position.

(2) We can rectify a fractional compressor, by multiplying itself and all the members of its column by its denominator, which must then be annexed as an external divisor. (See example 1, column 3.)

(3) If there is a unit among the elements, by selecting it as chief element, we make the other elements of its row taken with the proper signs the compressors. (Example 2, stage 1.)

(4) If zeros occur as compressors or co-compressors, the elements of the columns or rows of the early stage are carried over unchanged into the new stage.

(5) Hence we learn that the common method of reducing a line of a determinant to zeros, excepting one element, and thus lowering its order, is only a particular case of the compression method (as in stages 2 and 3 of example 2).

Example 1.

$$\begin{array}{|rrrr|}
\multicolumn{1}{c}{} & \multicolumn{1}{c}{} & \multicolumn{1}{c}{(\times 5)} & \multicolumn{1}{c}{} \\
\underline{25} & -15 & 23 & -5 \\
\underline{-15} & -10 & 19 & 5 \\
\underline{23} & 19 & -15 & 9 \\
-5^{*} & \underline{5} & \underline{9} & \underline{-5} \\ \cline{2-4}
\multicolumn{1}{c}{} & \multicolumn{1}{r}{1} & \multicolumn{1}{r}{\frac{9}{5}} & \multicolumn{1}{r}{-1}
\end{array}
\div 5 = \frac{5}{5}
\begin{array}{|rrr|}
10^{*} & \underline{340} & \underline{-30} \\
\underline{-25} & -40 & 20 \\
\underline{42} & 132 & -14 \\ \cline{2-3}
\multicolumn{1}{c}{} & \multicolumn{1}{r}{-34} & \multicolumn{1}{r}{3}
\end{array}$$

$$= 10 \begin{vmatrix} 810 & -55 \\ -1296 & 112 \end{vmatrix} = 10(90720 - 71280) = 194400.$$

Here the term selected as chief (-5 at foot of 1st column) being in an even marginal position, appears as $+5$ when multiplied by the next determinant. In order to rectify the fractional compressor $9/5$ of the first determinant we multiply it and its column by its denominator 5 which is written above its column, and is also written as an external divisor.

The first unmarked element of the first determinant is -15: to it we add 1×25 the product of its compressor and its co-compressor, giving 10 as the first element of the new determinant. So for the next element of the same row, $5 \cdot 23 + 5 \cdot 9/5 \cdot 25 = 340$, giving the second element: and for the third $-5 + (-1)\, 25 = -30$.

Example 2.

$$\underset{\overline{\begin{array}{ccccc}0 & -2 & 0 & -3 & 0\end{array}}}{\begin{vmatrix} 1^* & \underline{0} & \underline{2} & \underline{0} & \underline{3} & \underline{0} \\ \underline{0} & 1 & 0 & 2 & 0 & 1 \\ \underline{1} & 0 & 2 & 0 & 1 & 0 \\ \underline{0} & 1 & 0 & 3 & 0 & 1 \\ \underline{2} & 0 & 1 & 0 & 3 & 0 \\ \underline{0} & 3 & 0 & 4 & 0 & 1 \end{vmatrix}} = \begin{vmatrix} 1 & 0 & 2 & \underline{0} & 1 \\ \underline{0} & \underline{0} & \underline{0} & -2^* & \underline{0} \\ 1 & 0 & 3 & \underline{0} & 1 \\ 0 & -3 & 0 & \underline{-3} & 0 \\ 3 & 0 & 4 & \underline{0} & 1 \end{vmatrix} = -2 \begin{vmatrix} 1 & \underline{0} & 2 & 1 \\ 1 & \underline{0} & 3 & 1 \\ \underline{0} & -3^* & \underline{0} & \underline{0} \\ 3 & \underline{0} & 4 & 1 \end{vmatrix}$$

$$= -6 \underset{\overline{\begin{array}{cc}-2 & -1\end{array}}}{\begin{vmatrix} 1^* & \underline{2} & \underline{1} \\ \underline{1} & 3 & 1 \\ \underline{3} & 4 & 1 \end{vmatrix}} = -6 \begin{vmatrix} 1 & 0 \\ -2 & -2 \end{vmatrix} = -6\,(-2) = 12.$$

The diagonal from -2, which was chosen as chief element of the second stage above, strikes an odd marginal place, and hence its sign does not change. The diagonal from -3, the chief element of the third stage, strikes an even place, and therefore the sign changes when -3 becomes an external factor.

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